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Substituting the value of R in Σ, expanding the binomials, and reducing we finally get,

$$\Sigma = \frac{\pi}{4} \cdot \frac{\Delta^2}{S^2} \left[\frac{1}{\sin A/2} + \frac{1}{\sin B/2} + \frac{1}{\sin C/2} - 2 + \sin A/2 + \sin B/2 + \sin C/2 \right].$$

Also solved by F. R. Morris, Horace Olson, G. W. Hartwell, J. W. Clawson, Paul Capron, and J. W. Cromwell.

484. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that when spheres of uniform size are packed in the closest possible manner there is, in the interior of the mass, about 26 per cent. of voids.

Solution by Laenas G. Weld, Pullman, Ills.

The given space may be divided into (equal) rhombic dodecahedrons. Spheres inscribed to these will be packed in the closest possible manner, since each has contact with twelve equal spheres. Now, the rhombic dodecahedron may be conceived as follows: Place a cube and a regular octahedron in such relation that each of the twelve edges of either figure is bisected at right angles by an edge of the other; then join the extremities of the edges so related, thus forming twelve rhombs, which are the faces of the figure in question. Each rhomb has for one of its diagonals an edge of the cube and for the other the corresponding edge of the octahedron. Taking the edge of the octahedron as 1, that of the cube related to it as above is equal to $\frac{1}{2}\sqrt{2}$. Moreover the radius of the inscribed sphere is equal to $\frac{1}{2}$, which is also the altitude of each of the twelve rhombic pyramids into which the dodecahedron may be resolved. The area of the base of each of these pyramids is one half the product of its diagonals, which is equal to $\frac{1}{4}\sqrt{2}$; whence the volume of the dodecahedron is

$$D = 12(\tfrac{1}{3} \cdot \tfrac{1}{2} \cdot \tfrac{1}{4}\sqrt{2}) = \tfrac{1}{2}\sqrt{2}.$$

The volume of the sphere is $S = \pi/6$.

The ratio S/D is equal to the ratio of the aggregate volume of the spheres to that of the space in which they are packed, since this space is completely filled by the circumscribing rhombic dodecahedrons. This ratio is $0.7405-$. The voids, therefore, occupy $0.2595+$ of the space, or about 26 per cent.

Also solved with slightly different results by Paul Capron, G. Paaswell, J. W. Clawson, and Herbert N. Carleton.

CALCULUS.

400. Proposed by H. S. UHLER, Yale University.

The axis of a prism whose right section is a regular polygon of apothem a and n sides passes through the center of a sphere of radius R. Show that, in general, the volume may be expressed by the formula:

$$V = \tfrac{4}{3}\pi R^3 + \tfrac{2}{3}a^2 n \left(R^2 - a^2 \sec^2 \frac{\pi}{n} \right)^{1/2} \tan \frac{\pi}{n}$$

$$+ \tfrac{1}{3}an(3R^2 - a^2) \sin^{-1} \left[\frac{2a\left(R^2 - a^2 \sec^2 \frac{\pi}{n} \right)^{1/2} \tan \frac{\pi}{n}}{R^2 - a^2} \right] - \tfrac{4}{3}nR^3 \sin^{-1} \left[\frac{R \sin \frac{\pi}{n}}{(R^2 - a^2)^{1/2}} \right].$$

Also discuss the special cases when $a = R \cos(\pi/n)$ and when $n = \infty$.

Solution by R. K. Morley, Worcester, Mass.

Using cylindrical coördinates

$$V = 4n \int_0^{\pi/n} \int_0^{a \sec \theta} \int_0^{\sqrt{R^2-}} \rho\, dz\, d\rho\, d\theta = -\tfrac{4}{3}n \int_0^{\pi/n} (R^2 - a^2 \sec^2 \theta)^{3/2} d\theta + \tfrac{4}{3}n \int_0^{\pi/n} R^3 d\theta$$

$$= -\tfrac{4}{3}n \int_0^{\pi/n} (R^2 - a^2 \sec^2 \theta)(R^2 - a^2 \sec^2 \theta)^{1/2} d\theta + \tfrac{4}{3}\pi R^3.$$